

Comment on “Quasienergy anholonomy and its application to adiabatic quantum state manipulation”

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In their Letter [1], Tanaka and Miyamoto introduce a kicked spin model, for which they point out the generic existence of exotic eigenvalue anholonomy. They proceed to show the potential utility of the model for quantum state manipulation and quantum information processing.

In our view, there are three missing elements in the article, without which their discussion seems to be incomplete and the prospect of the application limited.

The first point is related to the different dynamical phase accumulated to different states during the variation of parameter λ . This would break the phase coherence of a superposed state which is required to ensure quantum parallelism in quantum computation. The cyclic nature of the parameter variation in the model comes in for rescue, however. For two level systems, the difference of accumulated dynamical phase between two states $|0\rangle$ and $|1\rangle$ after the cyclic variation of λ is $\int_0^{MT} (E_1 - E_0) dt = \int_0^{2\pi} (E_1(\lambda) - E_0(\lambda)) v_\lambda^{-1} d\lambda$ where $v_\lambda = \frac{d\lambda}{dt}$ is the velocity of the variation of λ , T the period of kick, M the number of kicks to complete a λ -cycle. Since v_λ is a quantity at our disposal, which we may set as a constant $v_\lambda = 2\pi/(MT)$ during the variation of λ , it could be used to ensure the phase difference becoming an integer multiple of 2π so that the phase coherence between the two states is intact. Possible modifications of above estimate by step-by-step variation of λ will not change the essential story line that the dynamical phase difference is controllable by proper choice of M .

The second point is on the proper consideration of Mead-Berry connection hidden behind the scene. Consider a *one-dimensionally mobile kicked spin* described by the Hamiltonian

$$\mathcal{H} = \frac{1}{2M} P^2 + \frac{\pi}{2} \sigma_3 + \frac{R}{2} (1 - \sigma_y) \sum_{m=-\infty}^{\infty} \delta(t - mT) \quad (1)$$

with $P = -i\frac{d}{dR}$. The second and the third term combined give exactly the original kicked spin model as appeared in FIG. 1 of [1] with R , the spatial coordinate variable, replacing the parameter λ as the coupling strength. First term represents the kinetic energy of the spin moving along the coordinate R . During a period $t = T$ (which we now set to be 1 for brevity), the evolution is described

by the unitary operator

$$U = e^{-i\frac{1}{2M}P^2} [e^{-i\frac{\pi}{2}\sigma_3} e^{-i\frac{R}{2}(1-\sigma_y)}]. \quad (2)$$

After diagonalizing the spin part of the system inside the bracket in the manner of [1], $e^{-i\frac{\pi}{2}\sigma_3} e^{-i\frac{R}{2}(1-\sigma_y)} \phi_s(R) = e^{-iE_s(R)} \phi_s(R)$, we write the eigenfunction Ψ of the total Hamiltonian \mathcal{H} as $\Psi = \sum_s \psi_s(R) \phi_s(R)$, and obtain the quasienergy equation

$$\sum_{s'} \left[e^{-i\{\frac{1}{2M}(\mathbf{P}-\mathbf{A})^2 + \mathbf{E}\}} \right]_{ss'} \psi_{s'}(R) = e^{-i\mathcal{E}} \psi_s(R), \quad (3)$$

where $\mathbf{P}_{ss'} = -i\frac{d}{dR} \delta_{ss'}$, $\mathbf{E}_{ss'} = E_s(R) \delta_{ss'}$ are momentum and effective potential matrices, respectively, and \mathcal{E} the quasienergy of the total Hamiltonina \mathcal{H} . The gauge potential \mathbf{A} is given by

$$\mathbf{A}_{ss'}(R) = \langle \phi_s(R) | i\partial_R \phi_{s'}(R) \rangle, \quad (4)$$

which cannot be globally gauged out because of the eigenspace anholonomy. For the specific example of the model (1), it is given by $\mathbf{A}_{00} = \mathbf{A}_{11} = 0$, $\mathbf{A}_{01} = -\mathbf{A}_{10} = -i/4$. Diagonal elements of \mathbf{A} are, in fact, always zero for kicked spin 1/2 with rank-1 perturbation, guaranteeing the eigenspace anholonomy. Thus we have non-Abelian gauge structure here, just as in the case of degenerate eigenvector anholonomy of Wilczek and Zee [2].

The above derivation of non-Abelian gauge leads to our third and final point. In no place between (2) and (4), any assumption on the adiabaticity of R variable nor the concept of Born-Oppenheimer *approximation* invoked. Equation (3) is exact, finite and well-defined as long as s runs on finite dimension.

We conclude that, contrary to the view implied in [1], all the physics inherent in gauge potential obtained from degenerate Berry phase is replicated with models with eigenvalue anholonomy without any degeneracy, and that is achievable without the assumption of adiabaticity.

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[1] A. Tanaka and M. Miyamoto, Phys. Rev. Lett. **98**, 160407 (2007).

[2] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).